Design of Perforated Plates

This paper describes a method for calculating stresses and deflections in perforated plates with a triangular penetration pattern. The method is based partly on theory and partly on experiment. Average ligament stresses are obtained from purely theoretical considerations but effective elastic constants and peak stresses are derived from strain measurements and photoelastic tests. Acceptable limits for pressure stresses and thermal stresses in heat-exchanger tube sheets are also proposed.

Introduction

The calculation of stresses in perforated plates is a subject which has received considerable attention as a result of the widespread use of flat tube sheets in heat-exchange equipment. Major contributions have been made by Horvay [1, 2], Malkin [3], Gardner [4, 5, 6], Duncan [7], Miller [8], Galletly and Snow [9], and Salerno and Mahoney [10]. Most of the published work has been limited to perforations arranged in an equilateral triangular pattern, and the present paper is no exception. The Pressure Vessel Research Committee of the Welding Research Council is currently sponsoring work on square patterns of holes but no results are available as yet.

Most heat-exchanger tube sheets are designed to meet the standards set by the Tubular Exchanger Manufacturers Association [11]. In these TEMA standards the thickness required to resist shear depends on the ligament efficiency of the perforations for the degree of Doctor of Philosophy.

Most of the proposed methods for analyzing perforated plates have involved the concept of an “equivalent” solid plate [3, 4]. In one method the equivalent solid plate has the same dimensions as the actual plate but its flexural rigidity is reduced by a factor called its deflection efficiency. In another method the equivalent plate is the same as the solid plate, but it has fictitious elastic constants E* and ν* in place of the actual constants of the material E and ν. The latter concept is used in this paper.

General Method of Analysis

The general method of evaluating stresses and deflections in a perforated plate having a triangular penetration pattern is:

Step 1. Calculate the nominal bending and membrane stresses and deflections of an equivalent solid plate having the effective modified elastic constants E* and ν* and the same dimensions as the perforated plate.

Step 2. Calculate physically meaningful perforated-plate stress values from the nominal stress values in the equivalent solid plate from Step 1. Deflections of the perforated plate are the same as the deflections of the equivalent solid plate.

When the perforated plate is part of a structure, as in the case of a heat exchanger, Step 1 is accomplished using classical structural-analysis methods. A study of the effective elastic constants for use in Step 1 is contained herein, and values based on

Nomenclature

Material Properties

- D* = E* H*/12 (1 - ν*²), effective flexural rigidity of perforated plate, lb-in.
- E = elastic modulus of solid material, psi
- E* = effective elastic modulus of perforated material, psi
- Sₘ = allowable membrane stress intensity of material, psi
- ν = Poisson’s ratio of material, dimensionless
- ν* = effective Poisson’s ratio for perforated material, dimensionless
- νₚ = Poisson’s ratio of plastic-model material, dimensionless
- νₚ* = effective elastic modulus of perforated plastic models, dimensionless
- αₚ = thermal expansion coefficient, in/in deg F

Co-ordinates and Dimensions

- r = radial distance of ligament from center of circular perforated plate, in.
- x, y, z = co-ordinates shown in Figs. 7 and 8, in.
- b = width of plate rim, Fig. 15, in.
- t = minimum ligament width, Fig. 6, in.
- tᵥᵥ = minimum ligament width for thin ligament at mis-drilled holes, in.
- rₒ = outside radius of plate rim, Fig. 13, in.
- dₒ = distance between center lines of perforations, Fig. 6, in.
- ρ = radius of perforations, Fig. 6, in.
- H = plate thickness, in.

Stresses

- σᵥᵥ = radial and tangential stresses in equivalent solid circular plate, psi
- σ₁ = σ, or σᵥᵥ, whichever has largest absolute value, psi
- σᵥᵥ = stresses in minimum ligament section, Fig. 7, psi

Discussion on this paper will be accepted at ASME Headquarters until January 10, 1962
Theoretical modified Malkin bending corrected for constrained warping by Sideros and Mahoney [10].
6 Experimental Sampson plane stress [13].
7 Experimental Sampson bending [13].

Sampson Effective Elastic Constants

The Sampson experimental values of the effective elastic constants for both plane stress and bending loads were obtained in tests on rectangular coupons at the Westinghouse Research Laboratories. The test specimens were made of plastic material, particularly E* and ρ* which are too low are used in the analysis, the theoretical rotation at the periphery of the tube sheet due to pressure loads across the tube sheet will be greater than the actual rotation. The calculated stresses at the periphery will then be lower than the actual stresses. This can be seen from fig. 26 of reference [12].

Correspondingly, if an effective elastic modulus which is too high is used in the analysis, the calculated pressure stresses at the center of the tube sheet would be low. If the tube sheet is taken to be too rigid, the calculated stresses, due to a pressure drop across the tube sheet, in the head and shell at their junction with the tube sheet would be lower than the actual stresses. Since stresses in these areas are usually among the highest stresses in a heat exchanger, it is important that they be evaluated properly.

Taking the tube sheet to be too flexible causes calculated thermal stress values in the tube sheet and in the remainder of the heat exchanger to be below the actual stress values. From the foregoing discussion it may be concluded that it is not possible to insure conservatism in heat-exchanger or tube-sheet stress calculations by assuming effective elastic constants which are known to be either too high or too low. These estimates of E* and B*, rather than the highest or lowest estimates should be used.

Many different sets of effective elastic constants for perforated materials having a triangular penetration pattern have been proposed. Five of the best known sets of values have been obtained from theoretical considerations and two have been obtained experimentally:

1 Theoretical Horvay plane stress [1].
2 Theoretical Horvay bending [2].
3 Theoretical modified Horvay bending, corrected for constrained warping by Salerno and Mahoney [10].
4 Theoretical Malkin bending [3].

Nomenclature

\( \theta_p, \theta_v, \theta_{gr} \) = stresses averaged through depth of plate, psi

\( \theta_{ns}, \theta_{ny} \) = transverse shear stress averaged through depth of plate, psi

\( \sigma_{nor} \) = nominal bending plus membrane stress at inside of rim, psi

\( \sigma_f \) = maximum principal stress based on average stresses across minimum ligament section, psi

\( \sigma_{int} \) = stress intensity based on stresses averaged across minimum ligament section at plate surface, psi

\( \sigma_{max} \) = maximum local stress, psi

\( \Sigma_{int} \) = stress intensity based on stresses averaged across minimum ligament section and through depth of plate, psi

\( K_c \) = value given in Fig. 13

\( K_m \) = value given in Fig. 15

\( K_p \) = value given in Fig. 10 for \( \beta = 0 \)

\( Y \) = value given in Fig. 12

\( P \) = normal force carried by ligament, kips/in.

\( \gamma \) = shear force carried by ligament, kips/in.

\( M \) = moment carried by ligament, kips-in.

\( F \) = pressure on plate surface under consideration, kips/in.

\( \Delta P \) = pressure drop across tube-sheet, psi

\( T_p \) = temperature at primary tube-sheet surface, deg F

\( T_s \) = temperature at secondary tube-sheet surface, deg F

\( T_{op} \) = temperature of hot side of tube sheet, Fig. 14, deg F

\( \beta \) = \( \sigma_f/\sigma_p \) or \( \sigma_f/\sigma_t \), whichever gives \( 1 \leq \beta \leq 1 \)

\( \psi \) = angular orientation of ligament, rad
values for the effective elastic constants for a plate in bending should approach the plane stress values as the plate gets thick. Fig. 1 shows the variation of $E^*$ with the relative thickness of a plate in bending, and Fig. 2 shows the same variation for $\nu^*$. Note the rather abrupt transition in the $E^*/E$-values that occur in the vicinity of $H/R = 4$. This appears to be what might be interpreted as a transition region between "thick" and "thin" perforated plates.

Obviously, it would be inconvenient to use one set of elastic constants for bending loads and another set for in-plane loads. Fortunately, this is not necessary as long as the plate is thicker than about twice the pitch of the perforations ($H/R > 4$) and this situation occurs in most heavy-duty heat-exchange equipment which requires the refined analysis described here. The effective elastic constants in bending for $H/R > 4$ do not differ greatly from the plane-stress values. Fig. 3 shows the bending constants at $H/R = 7$ plotted with the uniaxial plane-stress constants. Accordingly, the plane-stress constants appear to be the most acceptable values for plates having a relative thickness $H/R > 4$.

Notice that the uniaxial plane-stress values of effective Poisson's ratio ($\nu^*$ and $\nu^*_{xy}$) vary with the orientation of the load with respect to the hole pattern. The impracticality of factoring this anisotropic behavior into the analysis is immediately evident, and values must be used which represent the approximate Poisson's effect in all directions. This is not considered to be a serious problem, however, partly because the principal stresses are generally not oriented in the directions resulting in the largest differences between the effective Poisson's ratios (the $z$ and $g$-directions, respectively, in Fig. 3), and partly because these differences do not have a large effect on the calculated stresses.

Sampson evaluated the effective elastic constants for perforated plastic materials ($\nu_p = 0.5$) over a wide range of ligament efficiencies under bending and plane-stress loads. He then proceeded to evaluate the effect of material Poisson's ratio on the effective elastic constants. This was accomplished by measuring...
the effective elastic constants of an aluminum specimen \( (\nu = 0.327) \) in pure bending. The specimen had a relative thickness in the range of "thick plates" \( (H/R > 4) \). Hence, the test values obtained from this specimen are felt to be applicable in the entire range of parameters \( (H/R > 4) \), and for plate-stress loads as well as bending loads. Based on these test values, correlations were established on an empirical basis to estimate values of the effective elastic constants for any material and for any ligament efficiency. This relation is given in Fig. 4. The maximum deviation of any of the aluminum-bar test points from this empirical relation is 7 per cent. The corresponding relation between \( r^* \) for steels \( (\nu = 0.3) \) and \( r^* \) for plastic \( (\nu = 0.5) \) was used to modify the Sampson plane-stress \( r^* \)-values obtained in tests on plastic specimens in order to obtain corresponding values applicable to metal plates. The resulting values of \( r^* \) for \( \nu = 0.3 \) are recommended for use in design calculations. These values are given in Fig. 5. They can be used for both plane stress and bending loads in the plate, as discussed previously.

The effective elastic-modulus ratios \( E^*/E \) were found to be unaffected by changes in the Poisson's ratio of the material. Hence, the Sampson plane-stress values of \( E^*/E \), taken from Fig. 3, are recommended for use in design calculations. These values are also given in Fig. 5.

The smallest ligament efficiency of the coupons tested by Sampson was 15 per cent. Hence, the values given in Fig. 5 should not be extrapolated much below this value.

The error in stress values calculated using the general effective elastic constants given in Fig. 5 instead of the constants measured by Sampson (which depend on the type of loading, direction of loading, and the thickness of the plate) was evaluated. The largest error in the maximum local stresses or in the maximum average ligament stresses that are limited by the design criteria recommended herein for any type or direction of loading and any plate thickness \( (H/R > 4) \) was found to be 8 per cent.

**Stiffening Effect of Tubes**

When tubes are rolled or welded into a tube sheet, the question always arises regarding the degree to which the tubes increase the stiffness of the plate. As mentioned previously, it is not always conservative to assume either a maximum or a minimum value for the stiffness. In some strain-gage tests by A. Lohmeier, of the Westinghouse Steam Division, on a steam generator which had seen considerable service, very good correlation was obtained between calculated and measured stresses when full credit was taken for the tube wall in the calculation, but that, is, when the hole size was taken as the ID rather than the OD of the tube [16]. When the ligament efficiency was calculated on the basis of the OD of the tube, the measured stresses due to pressure loading averaged about 75 per cent lower than the calculated values. While this one test cannot be considered as conclusive evidence, the authors believe that it is a strong indication. Furthermore, it can be shown that since the membrane stresses in the tube sheet are usually low, very little residual compression is required in the tube wall to make it follow the strains in the drilled hole. Therefore the authors tentatively recommend that full credit be taken for the tube-wall thickness. Further confirmatory tests are planned.

**Proposed Stress Limits**

Before proceeding to the detailed calculation of stresses, it is necessary to decide which stresses are significant and, consequently, should be calculated and limited in order to assure an adequate design. The peak stress in a perforated plate is not necessarily the most significant one. Primary stresses, those which are required to satisfy the simple laws of equilibrium of internal and external forces, and which are consequently not self-limiting, should be the ones most severely limited. Secondary stresses, those which are only required to accommodate to an imposed strain pattern (e.g., thermal expansion) can be allowed to go higher than primary stresses. If the latter are kept lower than twice the yield strength, loadings subsequent to the initial loading will produce strains within the elastic limit. Peak stresses in localized regions are of interest only if they are repeated often enough to produce fatigue. For tube sheets, consideration must also be given to distortion of the holes which may cause leakage around the tube. The use of the maximum-shear theory of failure rather than the maximum-stress theory of failure is recommended. In order to
make allowable shear-stress values comparable to the more familiar tensile values, calculated stresses are expressed in terms of two times the maximum shear stress, which is the largest algebraic difference between any two of the three principal stresses. This quantity is called the "equivalent intensity of combined stress," or more briefly, the "stress intensity."

The following stress limits are proposed:

1 Typical Ligament in a Uniform Pattern
   (a) Mechanical Loads (i.e., pressure loads but not thermal loads):
      (i) The stress intensity based on stresses averaged across the minimum ligament section and through the thickness of the plate should be limited to prevent stretching of the plate. This stress is analogous to the average stress intensity in the shell of a pressure vessel under internal pressure and, consequently, should be limited to a value about the same as the allowable stress values in the ASME Boiler Code. (The question of whether or not the values in the 1959 edition of the Code are too conservative for vessels which are analyzed carefully for high stress is beyond the scope of this paper. In the 1959 Code, the allowable stresses do not exceed 5/8 of the yield strength of a ferrous material or 2/3 of the yield strength of a nonferrous material.) Let us call this basic stress intensity allowance $S_m$.
      (ii) The stress intensity based on stresses averaged across the minimum ligament section but not through the thickness of the plate should be limited to prevent excessive deflection. This stress is the sum of membrane plus bending effects and, since the limit-design factors for flat plates are greater than 1.5, it can safely be allowed to reach a value of 1.5 $S_m$.

(b) Combined Mechanical and Thermal Loads:
   (i) The stress intensity based on stresses averaged across the minimum ligament section but not through the depth should be limited to 3 $S_m$.
   (ii) The peak stress intensity at any point due to any loading should be limited by cumulative fatigue considerations, as described in [17].

2 Isolated or Thin Ligament. If a high stress occurs in a single ligament due to a misdrilled hole, the foregoing limits may be relaxed. For combined pressure and thermal loads, the stress intensity based on average stresses in the ligament cross section should be limited to 3 $S_m$, and peak stresses must still, of course, be subject to fatigue evaluation.
From the foregoing we see that three stress intensities should be calculated:

1. Average in ligament cross section, called $S_{ave}$
2. Average across ligament width at plate surface, called $\sigma_{ave}$
3. Peak, called $\sigma_{max}$

**Analysis of Ligament Stress Intensities**

Expressions for the average ligament stress intensities, limited by the design criteria suggested in the foregoing, are derived in this section from purely theoretical considerations. The analysis is quite general and can be used for any biaxiality condition of the stress field in the equivalent solid plate, and for any ligament orientation in the stress field. The accuracy of simplifying assumptions used in the analysis is examined using photoelastic test results. The analytical results are simplified and presented in a form suitable for design calculations.

In the concept of an equivalent solid plate, as considered herein, stresses and deflections of a solid plate having the effective elastic properties of the perforated material are evaluated. There is a unique state of stress within a body having a given set of elastic properties and subject to a particular load. Therefore, the stress field in an equivalent solid plate is the same as the stress field in the perforated plate on the same macroscopic scale for which the effective elastic constants were evaluated. Hence, the resultant loads carried by ligaments (at any arbitrary depth in the tube sheet) at any particular location must be equal to the resultant load carried by the equivalent solid plate. This is the basis of the analytical approach presented herein.

In perforated plates such as tube sheets, the perforations and ligaments are quite small relative to the over-all dimensions of the plate itself. As a result, the rate of change of the tangential and radial stresses with radial position in the equivalent solid plate (given by classical circular-plate theory) is small relative to the perforations. Hence, it is assumed that there exists only a negligible variation of load from any ligament to its adjacent parallel ligaments. Under these conditions, there are no side-sway bending moments in the minimum ligament sections. This can be seen by considering the equilibrium of an arbitrary cut at the surface, or at any arbitrary depth of the plate, as shown in Fig. 6. The stress field in the equivalent solid plate is given by $\sigma_r$ and $\sigma_\theta$ where the radial and tangential directions are principal directions in the equivalent solid plate. This stress field must be carried by the minimum ligament sections. Since there is no variation of stress from hole to hole, no net moment is supported by the cut section. Hence, the moments in the minimum ligament sections $M$ must be zero. Since the orientation of the cut is arbitrary, it is apparent that the side-sway moments $M$ are zero in all minimum ligament sections.

Yielding would tend to produce a uniform distribution of stress across the minimum ligament sections. Hence, in this analysis a three-dimensional element, subject to the average shear and tensile stresses in the minimum ligament section, is analyzed in order to evaluate the average stress intensities which are limited by the proposed design criterion.

**Analysis of Average Ligament Stress Intensities at Surfaces of Plate**

Having the principal stresses $\sigma_r$ and $\sigma_\theta$ at either surface of the equivalent solid plate, the problem of evaluating loads in the minimum ligament sections becomes statically determinate. The resultant load carried by the ligaments must be equal to the resultant load carried by the equivalent solid plate. The loads carried by the ligaments, as shown in Fig. 6, are then given by

$$ F = 2(\sigma_r \cos \psi) h \cos \phi + 2(\sigma_\theta \cos (\phi - \pi/2)) h \cos (\phi - \pi/2) $$

$$ V = 2(\sigma_r \sin \psi) h \cos \phi + 2(\sigma_\theta \sin (\phi - \pi/2)) h \cos (\phi - \pi/2) $$

Hence, the average stresses in a ligament at an arbitrary angle $\psi$ with the principal directions of the equivalent solid plate stresses $\sigma_r$ and $\sigma_\theta$ (as shown in Fig. 7) are given by

$$ (\sigma_{\psi \psi})_{ave} = \frac{1}{2h} \int_{-h}^{h} (\sigma_r \cos^2 \psi + \sigma_\theta \sin^2 \psi) \, dz \quad (3) $$

and

$$ (\tau_{\psi \theta})_{ave} = \frac{1}{2h} \int_{-h}^{h} \tau_{\psi \theta} \, dz = \frac{h}{h} (\sigma_r - \sigma_\theta) \sin \psi \cos \phi \quad (4) $$

In order to specify completely the state of stress in a minimum ligament section and to evaluate the ligament stress intensities (maximum-shear stresses) that are limited by the design criterion, something must be known about the stresses transverse to the ligament at the minimum ligament section $\sigma_r$. A three-dimensional view of a ligament is shown in Fig. 8(a). The average stresses acting on an element at a surface of the plate are shown in Fig. 8(b). The three-dimensional Mohr circle based on these average stresses, given by equations (3) and (4), is shown in Fig. 9. The Mohr circle, assuming zero transverse stress $\sigma_{\psi \psi}$, i.e.,
(b) AVERAGE STRESSES AT PRIMARY
OR SECONDARY SURFACE

(c) AVERAGE STRESSES AVERAGED
THROUGH DEPTH

Fig. 8 Three-dimensional stresses

ACTUAL STRESSES IN PLANE OF
TUBE SHEET

ACTUAL STRESSES IN PRINCIPAL
TRANSVERSE PLANES

CALCULATED STRESSES IN PLANE OF
TUBE SHEET ASSUMING PLANE STRESS

plane stress, is also shown for the plane of maximum shear. For
purposes of this analysis, the transverse stresses $\sigma_z$ will be taken
equal to zero. The significance of this important assumption will
be explained subsequently. The corresponding maximum prin-
cipal stress, based on the average value of the stresses across the
minimum ligament section, is given by

$$\sigma_{max} = \frac{R}{h} \left\{ \sigma_t \cos^2 \psi + \sigma_s \sin^2 \psi \right\}$$

where

$$\sigma = \frac{P}{K} \left[ (\sigma_x \cos^3 \psi + \sigma_s \sin^3 \psi) \right]^{1/2} + (\sigma_t - \sigma_x) \cos^3 \psi \sin^3 \psi^{1/2} \right\}$$

The comparable expression for the stress intensity (twice the
maximum shear stress) in the minimum ligament section is given by

$$\sigma_{int} = \frac{P}{h} \left[ (\sigma_x \cos^3 \psi + \sigma_s \sin^3 \psi) \right]^{1/2} + (\sigma_t - \sigma_x) \cos^3 \psi \sin^3 \psi^{1/2} \right\}$$

where $\sigma_{int}$ is the stress intensity limited by the design criterion.

Equation (6) gives the stress intensity based on the average
stress across any particular minimum ligament section for any
ligament orientation $\psi$ at either surface of the plate.

Consider the significance of assuming a zero transverse stress
at the minimum ligament section. Obviously, the transverse
stress must be zero at the edges of the minimum ligament section.
Moreover, this stress is usually small, even at the center of the
ligament. Photoelastic tests [18] have shown that the average
transverse stress usually has the same sign as the average longi-
tudinal stress, as shown in Fig. 9. When these stresses have the
same sign, the calculated value of the stress intensity in the plane
of the plate, based on stresses averaged across the minimum
ligament section, will always be equal to or greater than the
correct value of the stress intensity in that plane. This is il-
lustrated in Fig. 9.

There are conditions for which the maximum shear does not
occur in the plane of the plate. This happens when the minimum
principal stress in the plane of the plate has the same sign as the
maximum principal stress in that plane (the transverse shear
stresses being zero at the surfaces). The maximum shear can then
be found by rotating the element in the principal plane perpen-
dicular to the plate because the difference between the maximum
principal stress and the zero $Z$-direction stress\(^4\) is greater than
the difference between any other principal stresses. However,
the maximum shear stresses in the plane of the plate calculated by

\(^4\) The $Z$-direction stress due to pressure acting at the surface of a
plate is attenuated a short distance from the surface in the manner of a
bearing stress. Hence, although this stress should be considered in
the fatigue analysis of local peak stresses, it need not be considered in
the average stress-intensity limitations because the latter are only
intended to prevent excessive yielding and deformation.
assuming plane stress are always equal to, or greater than, the actual maximum shear stresses in any other plane. This can be seen by again considering the actual three-dimensional Mohr circle, as sketched in Fig. 9. However, it is not necessary to write equations for the shear stresses in planes other than the plane of the plate, provided that zero transverse stress, \( \sigma_z \), is assumed at the minimum ligament section.

At the edge of a perforated plate the stress field in the equivalent solid plate is isotropic. Hence, as indicated by equation (4), there are no shear stresses \( \tau_{xy} \) acting at the minimum ligament section. The maximum shear stress in this case (found by rotating the element that produced the described) acts on a plane at 45 deg to the plane of the plate. The theoretical expression for the maximum shear stress assuming plane stress in the minimum ligament section then gives the correct value for the actual maximum shear stress, even though the latter does not occur in the plane of the plate. Hence, the theoretical approach used herein gives the exact values of average stress intensities in ligaments near the center of a circular perforated plate regardless of the magnitude of the transverse stresses in the minimum ligament sections.

At the edge of a circular plate, however, high stresses may exist at the minimum ligament section. From the considerations, the maximum shear stress occurs in the plane of the plate, as illustrated in Fig. 9. The equation for the average stress intensity across the minimum ligament section, equation (6), then gives values which are higher than the actual values for many ligament orientations because of the assumption of zero transverse stress in the ligaments. The significance of this error was evaluated by assuming use of measured values of the transverse stress \( \sigma_t \) obtained photostatically by Sampson [18].

The error for a perforated plate under tensile loading having a ligament efficiency of 25 per cent and a minimum ligament width of 0.5 in., was evaluated. The maximum error for any biaxial condition and any orientation of the ligament in the stress field was found to be less than 3 per cent. This error increases with increasing ligament efficiencies. For a plate having a ligament efficiency of 50 per cent and a minimum ligament width of 0.5 in., the maximum error was found to be 5 per cent. These errors might tend to be greater for bending loads on relatively thin plates than for the tensile loads used in the photostatic tests. However, epoxy resin having a Poisson’s ratio of 0.5 was used in the photostatic tests and the resulting transverse stresses were probably higher than they would be for metals. Hence, the maximum error in the calculated stress-intensity values is probably no greater in a metal plate than the error evaluated herein from photostatic tests on plastic models.

The equation for the average stress intensity in the minimum ligament section, equation (6), may be simplified further for design calculations by considering the symmetry of the hexagonal array of neighboring holes surrounding the typical hole. It is apparent that the same stress distributions would result if the orientation of the ligaments were rotated \( \pm 60 \) deg in the equivalent solid-plate stress field, the actual stress distribution in the ligaments also being shifted \( \pm 60 \) deg. Consequently, at least two of the ligaments surrounding the typical hole pattern will be at most 30 deg rotated from that orientation which would produce the maximum stress intensity in the minimum ligament section. Near the center of a symmetrically loaded circular plate, the stress field is very nearly isotropic and the orientation of a particular ligament does not affect the stresses in that ligament appreciably. Near the periphery of a plate such as a tube sheet which contains a large number of holes, the angular orientation of the hole patterns with respect to the radius of the plate varies gradually around the periphery, encompassing the entire range of possible orientations. From these considerations, it is apparent that the expression for the stress intensity, equation (6), can be maximized

with respect to \( \psi \) for tube-sheet designs calculations without introducing undue conservatism. The resulting expression should be used to obtain stress intensities for typical ligaments in a uniform pattern, rather than for isolated ligaments. The expression for the orientation which gives the maximum stress intensity is given by:

\[
-\sigma_t \cos \psi = \frac{-\sigma_t \sin \psi + \sigma_t \cos \psi \cos \phi}{\cos \psi} + \frac{\sigma_t - \sigma_z}{\sigma_t + \sigma_z} \sin 2\phi \cos 2\psi = 0 \tag{7}
\]

From equations (6) and (7) it is possible to evaluate ligament stress intensities, maximized with respect to angular orientation in the stress field, for any ligament efficiency and any biaxiality condition. These equations can be written as functions of the biaxiality ratio \( \beta = \sigma_x/\sigma_y < \sigma_x/\sigma_z \), whichever gives \(-1 \leq \beta \leq 1\).

Equation (7), written in terms of \( \beta \), was used to find the orientation \( \psi \) which gives the maximum average ligament stress intensity in a stress field of biaxiality \( \beta \). This orientation was then used in equation (6) to evaluate the corresponding value of the average stress intensity. The resulting values are given by:

\[
\sigma_{av} = K \frac{R}{h} |\sigma_t| \tag{8}
\]

where \( \sigma_{av} \) = ligament stress intensity based on stresses averaged across minimum ligament section at either plate surface

\( R = \) value given in Fig. 10
\( K = \) reciprocal of ligament efficiency, Fig. 6
\( \sigma_t = \) \( \sigma_x \) or \( \sigma_y \) whichever has the largest absolute value.

(For example, if \( \sigma_t = -3000 \) psi and \( \sigma_x = 2000 \) psi, then \( \sigma_t = -3000 \) psi and \( |\sigma_t| = 3000 \) psi)

\( \sigma_{av} \) = stresses at either surface of equivalent solid plate obtained from Step 1 of analysis

To calculate ligament stress intensities based on stresses averaged across the width of the ligament but not through the depth of the plate, substitute the values of \( \sigma_x \) and \( \sigma_{av} \) at the surface of the plate into equation (8). The \( K \)-values for equation (8), given in Fig. 10, depend on the biaxiality of the stress field and vary with radial location in the plate. The resulting stress intensities will, of course, vary from one side of the plate to the other and will depend on the radial location in the plate.

Since equation (8) was developed by maximizing the stress intensity with respect to the angular orientation of the ligament, it may be overly conservative for plates having a small number of holes. As previously pointed out, the stresses near the center of the plate do not depend on the angular orientation of the ligament because the stress field is isotropic. However, it may be worth while to evaluate ligament stresses individually when the limiting value given by equation (8) occurs at the periphery of a plate having a small number of holes. Equation (6) gives the stress in a ligament having an arbitrary angular orientation \( \psi \).

Analysis of Ligament Stress Intensities Averaged Through Depth of Plate

The value of \( \sigma_x \) averaged through the depth of a plate at any location is equal to the value of \( \sigma_{av} \) averaged through the depth at that location. Moreover, these average values do not vary with location in a symmetrically loaded circular plate because they are produced by membrane-type loads. From equation (4), the average shear stress in the plane of the plate due to membrane
loads $T_{zr}$ is zero at the minimum ligament sections. The transverse shear stress averaged through the depth $T_z$ varies linearly with radial location $r$ in a circular plate.

Fig. 8(c) shows an element subject to the shear and tensile stresses averaged across the minimum ligament section and averaged through the depth of the plate. The three-dimensional Mohr circle based on these stress values is shown in Fig. 11. Since the transverse stress in the ligament $T_z$ has the same sign as the longitudinal stress $T_r$ (as previously discussed), it is apparent that the maximum shear stress due to membrane loads can be found by rotating an element in the principal plane subject to the transverse shear $T_{zr}$. The average stress intensity in a ligament at any radial distance $r$ from the center of the plate is given by

$$S_{eff} = \frac{R}{h} \left[ (\frac{\Delta P}{H})^2 + (\Delta T_{zr}/H) \right]^{1/2} \quad \text{(max with } r = \text{radius to outermost ligament)} \quad (9)$$

where

- $\Delta P$ = pressure drop across plate
- $r$ = radial distance of ligament from center of plate
- $\Delta T_{zr}$ = stresses averaged through depth of equivalent solid plate
- $H$ = thickness of plate

Peak Stresses in Perforated Plates

Maximum local stresses due to all loads (mechanical and thermal) are also limited by the suggested design criteria of this paper. These stresses can be evaluated from the known stresses in the equivalent solid plate using the stress multipliers obtained photoelastically by Sampson. A minor correction was made on these multipliers to account for the nonlinearity of the stress distribution.
The multipliers $Y$ are functions of the biaxiality of the stress field in the equivalent solid plate $\beta = \sigma_r/\sigma_t$ or $\sigma_\theta/\sigma_t$ (whichever gives $-1 \leq \beta \leq 1$). This ratio varies, of course, with radial location in the plate. The maximum stress for any particular thermal or pressure load is then given by the relation:

$$\sigma_{max} = Y\sigma_t + P$$

where

- $\sigma_t = \sigma_r$ or $\sigma_\theta$ (whichever has the largest absolute value)
- $Y$ = value given in Fig. 12
- $P$ = pressure acting on surface

All thermally induced maximum local stresses, as well as pressure stresses, must be considered in the cumulative fatigue limitations on the values of $\sigma_{max}$. The values given by equation (10) are the peak stresses throughout the perforated portion of the plate.

Most perforated circular plates have unperforated rims. Photoelastic tests on tube-sheet models have revealed the existence of high local stresses at the perforations adjacent to the rim (15). These peak stresses appear to be due to the influence of the rim and cannot be calculated by equation (10), but may be approximated by the expression

$$\sigma_{max} = K_r\sigma_{rim} + P$$

where

- $\sigma_{rim} =$ nominal bending plus membrane stress at inside of rim
- $K_r =$ value given in Fig. 13

$\sigma_{rim}$ is evaluated in Step 1 of the general analytical approach, the rim being treated as a plate or ring depending on its dimensions.

The $K_r$-values in Fig. 13 were derived from known values of stress concentration in a bar with a semicircular notch and were checked against the photoelastic results of Sampson and Leven."
Evaluation of Special Cases of Thermally Induced Stresses in Tube Sheets

Thermal "Skin Effect." In heat exchangers, the major part of the tube-sheet thickness is at the primary temperature by virtue of the perforations through which the primary fluid passes. The difference in temperature between the primary and secondary sides of the tube sheet occurs very near the secondary surface, resulting in what is commonly called a thermal skin effect. Because of the thermal film drop, the entire difference between the primary and secondary fluid bulk temperatures does not contribute to the skin effect. Credit may be taken for the temperature drop in the thermal boundary layer at the secondary side of the tube sheet when this drop can be evaluated. Stresses due to this effect are given by

\[ \sigma_{\text{max}} = K_D E a T (T_H - T_c) / (1 - \nu) \]  

\[ \sigma_{\text{max}} = K_D E a T (T_H - T_c) / (1 - \nu) \]

where
- \( K_D \) = stress-concentration factor from Fig. 14
- \( E \) = material properties of tube sheet
- \( a \) = thermal expansion coefficient, in/in(deg F)
- \( T_H \) = metal temperature at secondary tube-sheet surface, deg F
- \( T_p \) = primary temperature, deg F
- \( T_c \) = secondary temperature, deg F
- \( \nu \) = Poisson's ratio

Stresses for Temperature Drop Across Diametral Lane of U-Tube Type Steam-Generator Tube Sheet. In the case of a U-tube type steam generator, the unperforated diametral lane separates the inlet and outlet sides of the tube sheet, and large thermal stresses may arise because of a temperature difference between these sides. The resulting maximum local stresses in the ligaments of the tube sheet can be approximated by

\[ \sigma_{\text{max}} = K_F E a (T_p - T_c) / 2 \]

where
- \( K_F \) = uniaxial (\( \beta = 0 \)) stress multiplier from Fig. 12
- \( E^* \) = effective elastic modulus for tube sheet

The limited stress intensity in a particular out-of-tolerance ligament can then be evaluated by substituting the smallest ligament...
The maximum local stresses due to all loads (mechanical and thermal) in an isolated or thin ligament in a nominally uniform pattern are limited by fatigue considerations as the peak stresses in a typical ligament in a uniform pattern. The increase in the local stresses caused by the presence of a particular out-of-tolerance thin ligament was evaluated in photoelastic tests. The increase in peak stresses was found to be a function of the biaxiality of the stress field, and the direction of the displacement of the misdrilled hole with respect to the hole pattern, as expected. The variation of the increase in peak stresses with ligament efficiency was found to be small. The maximum increase in local stresses occurred when the hole was displaced at 30° to the line of hole centers. Using the results for this case, the maximum local stress in a thin ligament is given by

$$\sigma_{\text{max}} = K_w \sqrt{\psi} + P$$

where

- $\sigma_1 = \sigma_x$ or $\sigma_y$ (whichever has the largest absolute value)
- $K_w = \text{value given in Fig. } 15$
- $P = \text{pressure acting on surface}$
- $\psi = \text{value given in Fig. } 12$

The $K_w$-value given in Fig. 15 can be used for any ligament efficiency.

**Summary and Conclusions**

1. Effective elastic constants for both plane stress and bending loads for any plate thickness ($H/R > 4$) are given in Fig. 5.
2. A complete structural-design criterion for perforated plates is proposed. The limited stress values are summarized in the following table for a nominal ligament in a uniform pattern.

<table>
<thead>
<tr>
<th>Load</th>
<th>Stress intensity</th>
<th>Equation</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>Average across ligament at either surface of plate</td>
<td>(8)*</td>
<td>$1.5S_w$</td>
</tr>
<tr>
<td></td>
<td>Average across ligament and through thickness</td>
<td>(9)</td>
<td>$S_w$</td>
</tr>
<tr>
<td>Combined pressure and thermal</td>
<td>Average across ligament at either surface of plate</td>
<td>(8)*</td>
<td>$3S_w$</td>
</tr>
<tr>
<td>Cyclic pressure and thermal</td>
<td>Peak in ligaments</td>
<td>(10)</td>
<td>Cumulative fatigue</td>
</tr>
<tr>
<td>Cyclic thermal skin effect</td>
<td>Peak at surface</td>
<td>(12)</td>
<td>Cumulative fatigue</td>
</tr>
<tr>
<td>Cyclic thermal (temperature difference across diametral plane)</td>
<td>Peak at holes adjacent to diametral plane</td>
<td>(14)</td>
<td>Cumulative fatigue</td>
</tr>
</tbody>
</table>

* Equation (8) was obtained by maximizing the stress intensity with respect to the angular orientation of the ligament. If the plate contains only a small number of holes and if the limiting stresses occur at the periphery of the plate, a more accurate evaluation of this stress intensity which takes into account the angular orientation of the ligament may be justified. Equations (6) gives the corresponding stress intensity in a ligament with any angular orientation $\phi$. The maximum value of this stress intensity for all ligaments should be limited as indicated in the table.
3 A method of evaluating the acceptability of misdrilled holes is given. The relevant stresses and their proposed limits are given in the following table.

<table>
<thead>
<tr>
<th>Load</th>
<th>Stress limits</th>
<th>Equation</th>
<th>Limit</th>
<th>Cumulative fatigue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined pressure and thermal</td>
<td></td>
<td>Equation (9) with $h_0$ substituted for $h$</td>
<td>35 $p_a$</td>
<td>fatigue</td>
</tr>
<tr>
<td>Cyclic pressure and thermal</td>
<td></td>
<td>Peak in ligaments (15)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 The effective elastic constants and peak stress multipliers recommended herein are based on those obtained experimentally by Sampson. Stresses and deflections calculated using these values showed better agreement with the test results obtained by Leven on uniformly loaded, simply supported, circular perforated plates than any of the other approaches mentioned herein.

For most conventional steam generators, the design basis recommended herein allows a slightly thinner tube sheet than does TEMA [11]. For example, in a typical high-pressure design where TEMA requires a minimum tube-sheet thickness of 10 in., the design methods described herein require a minimum thickness of $9/8$ in. If $S_e$ is taken as $V_1$ of the yield strength of the material and full credit is taken for the tubes. On the other hand, where severe thermal loads are anticipated, it may be necessary to make design modifications in order to meet the criteria recommended herein, whereas TEMA does not account for thermal loads.

Acknowledgments

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References

3. See reference [14], figs. 7-11, and reference [15], figs. 18, 19, 20, 24, and 25.

13 Printed in U. S. A.